

THERMAL INTERACTION AT THE GAS - LIQUID  
INTERFACE IN AN APPARATUS WITH AN  
IMMERSED BURNER

A. N. Alabovskii

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An equation is derived for the ratio of the temperature drops in the boundary layers of gas and liquid; it is found that the temperature of the liquid at the phase interface is almost equal to that of the liquid in the apparatus.

The heat and mass transfer between water and moist air indicates that the thermal resistance of the liquid has very little influence on the rate of evaporation or condensation, and that this can usually be neglected [2], provided that the temperature difference is small and that there is correspondingly only a small difference in the mean heat flux densities from evaporative cooling and gas drying. If an apparatus has an immersed burner, the temperature of the exhaust gas may be 1600-1800°C, while the liquid in contact with it may be at 80-120°C. This large temperature difference makes it necessary to examine the thermal interaction at the interface when there is such a burner and to elucidate the role of the thermal resistance of the liquid.

When an aqueous solution evaporates, the heat  $q_\alpha$  transferred from the gas to the liquid is partly consumed in evaporation  $q_e$  and partly transferred to the liquid  $q_l$ , where it goes to heat the solution entering the apparatus, some of this heat thus being lost with the solution leaving the apparatus, and also via the wall of the apparatus to the surrounding medium:

$$q_\alpha = q_e + q_l.$$

We put  $q_e/q_l = m$  and also  $q_\alpha = \alpha''\theta_0''$  and  $q_l = \alpha'\theta_0'$ ; then the ratio of the temperature differences in the gas and liquid is

$$\frac{\theta_0''}{\theta_0'} = \frac{\alpha'}{\alpha''} (1 + m). \quad (1)$$

We may determine the heat-transfer coefficient between the gas and the solid wall on the usual basis that at the surface of the solid there is a thin immobile gas layer; heat passes through this only by thermal conduction. The dry heat-transfer coefficient in a laminar boundary layer during forced flushing of a flat solid wall is [3] defined by

$$\alpha = \frac{3}{2} \cdot \frac{\lambda}{k}. \quad (2)$$

It has been shown in studies of the mass-transfer mechanism at a gas-liquid interface that the normal component of the velocity is eliminated by surface-tension forces as material approaches the surface, and at the surface itself the material moves transversely solely as a result of molecular motion, much as in the motion of material near a solid [5]. This physical model applies also to heat transfer at a phase interface between liquid and gas [4].

The boundary conditions for the thermal boundary layer in a gas-liquid system coincide with the boundary conditions for such a layer at the surface of a solid wall, so we use the following form for the temperature distribution in the boundary layer [3]:

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$$\frac{\theta}{\theta_0} = 1.5 \frac{y}{k} - 0.5 \left( \frac{y}{k} \right)^3. \quad (3)$$

Equation (2) enables us to determine the heat-transfer coefficient at the liquid-gas boundary; one has to determine  $k$ , which is dependent on the thickness of the hydrodynamic boundary layer  $\delta$ , which itself in a liquid with an immersed burner is determined by the speed of a gas bubble relative to the liquid. Within a rising gas bubble there are tangential stresses at the phase interface, which give rise to a descending motion in the peripheral gas layers and result in a hydrodynamic boundary layer. We assume that the bubble is motionless and that the liquid flushes over it with a velocity  $w'_0$ , which is equal to the rise speed of the bubble. When a fluid passes by a solid wall, the thickness of the hydrodynamic boundary layer is determined by solving differential equations for the boundary layer subject to appropriate boundary conditions.

An approximate method is to use the momentum equation or the integral Karman relation for the planar steady-state flow in the boundary layer [4]:

$$\frac{d}{dx} \int_0^{\delta} \rho w_x^2 dy - w_0 \frac{d}{dx} \int_0^{\delta} \rho w_x dy = -\tau. \quad (4)$$

An additional relationship is the velocity distribution in the boundary layer, which is given by a polynomial of third degree:

$$w_x = a + by + cy^2 + dy^3. \quad (5)$$

We used this approximate method to determine the thickness of the hydrodynamic boundary layer for a planar interface between the moving liquid and the immobile gas; coefficients  $a$ ,  $b$ ,  $c$ , and  $d$  of (5) are derived from the boundary conditions, which differ from those for a gas on a solid wall.

The speed of the liquid at the phase interface is  $w'_b$ ; we put

$$(w'_x)_{y=0} = w'_b$$

For  $y = -\delta'$ , the velocity  $w'_x$  becomes equal to the velocity of the unperturbed flow  $w'_0$ , so

$$(w'_x)_{y=-\delta'} = w'_0.$$

The internal-friction force is here zero, so

$$\left( \frac{\partial w'_x}{\partial y} \right)_{y=-\delta'} = 0.$$

We assume that the tangential stresses are constant directly at the interface, so we have the fourth boundary condition

$$\left( \frac{\partial^2 w'_x}{\partial y^2} \right)_{y=0} = 0.$$

These boundary conditions correspond to the following velocity distribution in the boundary layer of liquid:

$$w'_x = -\frac{3}{2} \cdot \frac{w'_0 - w'_b}{\delta'} y + \frac{1}{2} \cdot \frac{w'_0 - w'_b}{(\delta')^3} y^3 + w'_b \quad (6)$$

The boundary conditions are put as follows for the hydrodynamic boundary layer:

$$\begin{aligned} (w''_x)_{y=0} &= w''_b; & (w''_x)_{y=\delta''} &= 0; \\ \left( \frac{\partial w''_x}{\partial y} \right)_{y=\delta''} &= 0; & \left( \frac{\partial^2 w''_x}{\partial y^2} \right)_{y=0} &= 0. \end{aligned}$$

Then we get the equation for the velocity distribution in the boundary layer of gas:

$$w''_x = -\frac{3}{2} \cdot \frac{w''_b}{\delta''} y + \frac{1}{2} \cdot \frac{w''_b}{(\delta'')^3} y^3 + w''_b. \quad (7)$$

We use (6) and (7) to find the tangential stresses at the phase interface:

$$\tau' = \mu' \left( \frac{\partial w'_x}{\partial y} \right)_{y=0} = -\frac{3}{2} \mu' \frac{w'_0 - w'_b}{\delta'}, \quad (8)$$

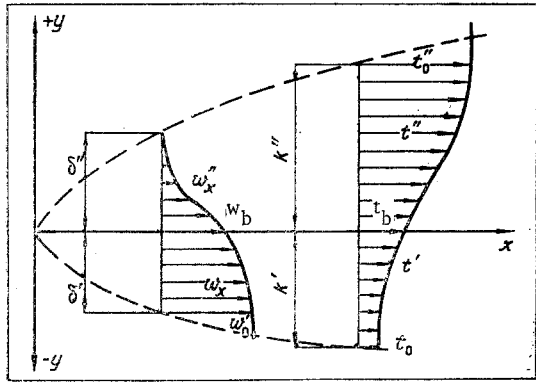


Fig. 1. Velocity and temperature in boundary layers of liquid and gas.

$$\tau'' = \mu'' \left( \frac{\partial w_x''}{\partial y} \right)_{y=0} = -\frac{3}{2} \mu'' \frac{w_b''}{\delta''}. \quad (9)$$

We substitute for  $w_x$  and  $\tau$  into (4) to get the thicknesses of the boundary layers of gas and liquid:

$$\frac{1}{(\delta'')^2} = \frac{11}{140} \cdot \frac{\rho'' w_b''}{\mu'' x}, \quad (10)$$

$$\frac{1}{(\delta')^2} = \frac{\rho' [66 (w_b')^2 - 27 w_0' w_b' - 39 (w_0')^2]}{840 \mu' (w_0' - w_b') x}. \quad (11)$$

From (10) and (11) we see that to determine the thickness of the hydrodynamic boundary layer we need to know the velocity at the boundary at the phase interface; the conditions for absence of slip between the phase enables us to put [4] that

$$w_b' = w_b'' = w_b.$$

Also, the condition for dynamic equilibrium at the phase interface is

$$\tau' = \tau''. \quad (12)$$

We substitute the corresponding  $\tau$  into (12) and find the equation for  $w_b$ :

$$39 (w_0')^3 - 12 (w_0')^2 w_b - 93 w_0' w_b^2 + 66 w_b^3 = -66 \frac{\mu'' \rho''}{\mu' \rho'} w_b^3. \quad (13)$$

We have  $\mu' \rho' \gg \mu'' \rho''$  for the gas-liquid system, so we can rewrite the equation in the form

$$39 (w_0')^3 - 12 (w_0')^2 w_b - 93 w_0' w_b^2 + 66 w_b^3 = 0. \quad (14)$$

It follows from the latter equation that  $w_b = w_0'$ , so an immobile gas has no influence on the speed of the adjacent layers of liquid, and the thickness of the hydrodynamic boundary layer at the gas-liquid interface is zero. The frictional forces cause the liquid to drag along the adjacent gas layer, and the gas speed at the phase interface becomes equal to the liquid speed (Fig. 1).

We find the thickness of the thermal boundary layer of liquid via the heat flow equation for the boundary layer [3]:

$$\frac{d}{dx} \int_0^k (\theta_0' - \theta') w_x' dx = a' \left( \frac{d\theta'}{dy} \right)_{y=0}. \quad (15)$$

Then  $\theta'$  is determined from (3) and  $w_x' = w_0'$ ; we substitute for  $w_x'$  and  $\theta'$  in (15) to get after integration that

$$(k')^2 = 8 \frac{a' x}{w_0'}. \quad (16)$$

We substitute (16) into (2) to get

$$\alpha' = \frac{3}{2} \lambda' \sqrt{\frac{w_0'}{8 a' x}} = 0.53 \lambda' \sqrt{\frac{v'}{a'}} \sqrt{\frac{w_0'}{v' x}}. \quad (17)$$

We take the value of the Prandtl number of the gas as being approximately unity [4], and in this case

$$k'' = \delta'' = \sqrt{\frac{140}{11} \cdot \frac{\mu'' x}{\rho'' w_0'}} = \sqrt{\frac{140}{11} \cdot \frac{v'' x}{w_0'}}. \quad (18)$$

From (2) and (18) we get

$$\alpha'' = \frac{3}{2} \lambda'' \sqrt{\frac{11}{140} \cdot \frac{w_0'}{v'' x}} = 0.42 \lambda'' \sqrt{\frac{w_0'}{v'' x}}. \quad (19)$$

From  $\alpha'$  and  $\alpha''$  we get with (1) that the ratio of the temperature differences in the gas and liquid is

$$\frac{\theta_0''}{\theta_0'} = \frac{\lambda'}{\lambda''} \left( \frac{v''}{v'} \right)^{0.5} \left( \frac{v'}{a'} \right)^{0.5} (1 + m). \quad (20)$$

We take the temperature of the burnt gases from the burner as 1200°C, while the water temperature in the apparatus is 90°C. The following are the physical parameters of the water and burnt gases corresponding to these temperatures:  $\lambda' = 0.68 \text{ W/m} \cdot \text{deg}$   $\nu' = 0.326 \cdot 10^{-6} \text{ m}^2/\text{sec}$ ;  $\nu'/a' = 1.95$ ;  $\lambda'' = 0.126 \text{ W/m} \cdot \text{deg}$ ;  $\nu'' = 221 \cdot 10^{-6} \text{ m}^2/\text{sec}$ .

We substitute these quantities into (20) to get

$$\frac{\theta_0''}{\theta_0'} = 246 (1 + m).$$

Here  $m$  is close to zero for heating of water, and the temperature at the gas-liquid interface is found as 94.5°C, while  $m > 0$  for evaporation of the water and can become infinite (adiabatic evaporation), so the temperature at the boundary approaches even more closely the temperature of the liquid in the apparatus. The transverse flow of evaporating water reduces  $\alpha''$ , which in turn increases the temperature difference in the gas [3]. We may also suppose that transition to a turbulent boundary layer will increase the temperature difference in the gas on account of the rise in the exponent to Reynold's number in the heat-transfer equations, and hence in the ratio  $\nu''/\nu'$  in (20). Studies of heat interaction at phase interfaces have been made in relation to planar surfaces; the conditions are really very different at the interface surfaces in an apparatus with an immersed burner, and so the results enable us to establish only a general picture for the interaction between the gas and liquid, though the following conclusions can be drawn.

1. The liquid temperature at the phase interface is practically equal to the temperature of the liquid in the apparatus, so the thermal resistance of the liquid can be neglected.
2. The hydrodynamic boundary layer produced within the gas bubbles is dependent on the liquid velocity at the interface and is governed by the rate of rise of the bubbles and by  $\rho''\mu''/\rho'\mu'$ ; as usually  $\rho'\mu' \gg \rho''\mu''$ , the viscosity and density of the liquid do not affect the thermal resistance of the gas phase, and hence also the resistance to heat transfer. These parameters can, however, affect the rate of heat transfer only to the extent to which they influence the area of the interface surface.
3. As all the temperature difference occurs in the boundary layer of gas, an apparatus with an immersed burner can be used to concentrate aqueous solutions of organic substances without danger of oxidation on account of contact with the hot gas. The latter result is confirmed by experimental studies [1] of the evaporation of sulfate solutions used in artificial silk manufacture.

#### NOTATION

$x, y$	are the coordinates (Fig. 1);
$\alpha$	is the heat-transfer coefficient;
$\lambda, a$	are the thermal conductivity and thermal diffusivity;
$\rho$	is the density;
$\nu$	is the kinematic viscosity;
$\mu$	is the dynamic viscosity;
$\tau$	is the tangential stress at phase interface;
$\theta = t - t_b, \theta_0 = t_0 - t_b$	are temperature differences;
$t$	is the temperature in boundary layer;
$t_b$	is the temperature at phase boundary;
$t_0$	is the temperature outside boundary layer;
$w_x$	is the speed in boundary layer;
$w_b$	is the speed at phase boundary;
$w_0$	is the speed outside boundary;
$k$	is the thickness of thermal boundary layer;
$\delta$	is the thickness of hydrodynamic boundary layer.

#### Superscripts

- ' denotes the liquid;  
 '' denotes the gas.

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